TUTORIAL SHEET 1

QUESTION 1 (a)

Define Failure Rate and hazard rate and explain their application to the reliability of components and repairable systems.

SOLUTION:

Failure rate: items which are repaired when they fail, reliability is the probability that failure will not occur in the period of interest, when more than one failure can occur. It can also be expressed as the rate of occurrence of failures (ROCOF)

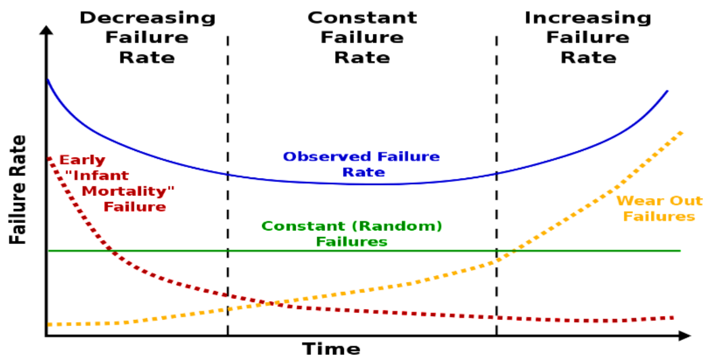
Hazard rate: For a non-repairable item (light bulb) reliability is the survival probability over the items expected life, or a period of its life, when only one failure can occur.

QUESTION 1 (b)

Discuss the "bathtub curve"

SOLUTION:

The combined effect generates the so-called bathtub curve, this shows an initial decreasing hazard rate or infant mortality period, an intermediate useful life and a final wear out period.



QUESTION 2

Explain the difference between reliability and durability and how they can be specified in a product development programme.

SOLUTION:

Reliability is typically defined as: The probability that an item will perform a required function without failure under stated conditions for a stated period of time.

Durability is an aspect of reliability related to the ability of an item to withstand the effects of time (distance travelled, operating cycles etc) dependent mechanisms such as fatigue, wear, corrosion and so on. Is associated with wear-out product and can be estimated for most products as usage is uncertain.

QUESTION 3 (a)

List the potential economic outcomes of poor reliability and identify which costs are directly quantifiable and which are intangible. Explain how they can be minimised and discuss the extent to which very high reliability (approaching zero failures) is achievable in practice.

SOLUTION:

Quantifiable : maintenance, spare parts, customer returns, loss of market share
Intangible : loss of faith of customers, loss of failure sales
Minimize by having a good quality assurance

QUESTION 3 (b)

What are the major factors that might limit the achievement of very high reliability?

SOLUTION:

Cost, human resource, quality suppliers, skill level of employees

QUESTION 4

After processing the existing programming cost data and running a regression model on the previous projects the cost of product development and manufacturing (CDM) has been estimated to follow the equation: CDM = €0.8 million +€3.83 million x R^2 (R is the achieved product reliability at service life and is expected to be above 90%). The cost of failure (CF) has been estimated as the sum of the fixed cost of €40,000 plus variable cost of €150 per failure. The total number of the expected failures is n x(1-R), where n is the total number of produced units.

Considering that the production volume is expected to be 50,000 units, estimate the optimal target reliability and the total cost of the programme.

SOLUTION:

Total cost= 800,000+3,830,000R^2+40,000+150(1-R)

N=50,000

df($\frac{total\ cost}{dR}$)=(2*3,830,000R)-150*50,000 =

R = 0.979

MATHS REFRESHER

1. Differentiate the following:

(i) $3e^{-2\lambda t}$

$$-6e^{-2\lambda t}\lambda$$

(ii) $3x^2+2x$

$$6x+2$$

2 Solve the following for t

$0.99=e^{-(\frac{t}{50})^2}$

$$t=5.01256$$

3. Express the following as a single fraction

$$1-\frac{b}{10^{-6}}$$

1-1000000b

4. Integrate the following

$$\int_0^\infty e^{-2\lambda t}\,dt$$

$$\frac{1}{2t}$$

5. Simplify the following:

$$\frac{6\lambda e^{-\lambda t}-6\lambda e^{-2\lambda t}+3\lambda^2 e^{-3\lambda t}}{3e^{-\lambda t}+9\lambda e^{-2\lambda t}+6e^{-4\lambda t}}$$

$$\frac{\lambda(2e\text{^}{-\lambda t}-2e\text{^}{-2\lambda t}+\lambda e\text{^}{-3\lambda t})}{e\text{^}{-\lambda t}+3\lambda e\text{^}{-2\lambda t}+2e\text{^}{-4\lambda t}}$$

6. Evaluate the following if t = 100 $\frac{3t^3}{10^5}$

TUTORIAL SHEET 2 (PROBABILITY THEORY)

QUESTION 1

The tensile strength of laminated composite plates (x) is measured in N/mm^2. If two events A and B are defined as A = {X|x>10.5}, B={X|x<15.0} . Describe the following events

i) $\overline{A}$

{X|x ≤ 10.5}

ii) $\overline{B}$

{X|x ≥ 15.0}

iii) A U B

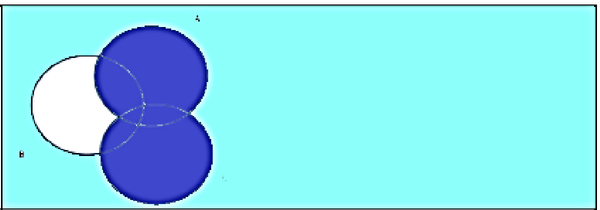
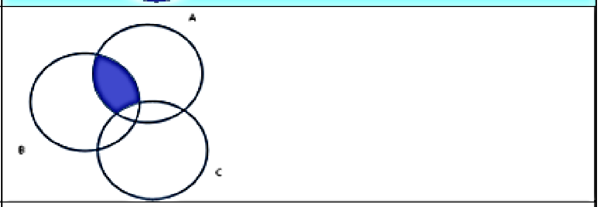
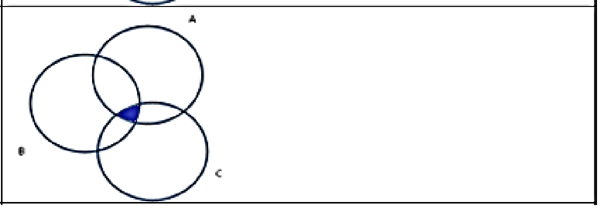
{X|x > 10.5} U {X|x < 15.0}

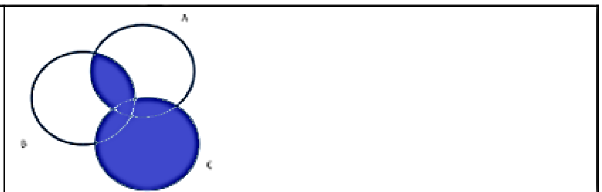
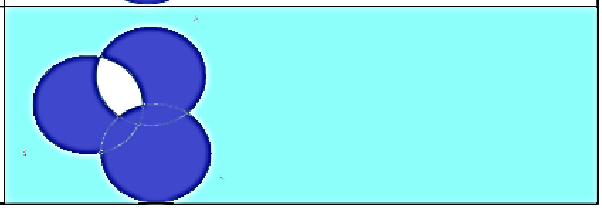
iii) A $\cap B$

{X|10.5 < x < 15.0}

QUESTION 2

The diagram below shows events, A, B and C. Show the following events by shading the appropriate region in the Venn diagram

$\overline{B}$	
$A \cap B$	
$A \cap B \cap C$	

$(A \cap B) \cup C$	
$\overline{(A \cap B)} \cup C$	

QUESTION 3

A machinist produced 100 shafts according to the specification 1 ±0.001.

During inspection, the diameters of 85 shafts were found to be within tolerance limits and 15 were found to be outside the tolerance band.

If 6 are randomly selected, find the probability of finding the diameter of at least one shaft falling outside the tolerance limits.

SOLUTION:

P(at least one out 6 is outside)

1-P(all 6 are good)

$$1-\left(\frac{85}{100}*\frac{84}{99}*\frac{83}{98}*\frac{82}{97}*\frac{81}{96}*\frac{80}{95}\right)$$

= **0.633**

QUESTION 4

In the test firing of a missile, there are some events that are known to cause the missile to fail to reach its target. These events are listed below, together with their approximate probabilities of occurrences during a flight.

Event	Probability
A1: Cloud Reflection	0.0001
A2 Precipitation	0.005
A3 Target evasion	0.002
A4 Electronic Countermeasures	0.04

The probabilities of failure if these events occur are:

P(F/A1) = 0.3, P(F/A2) = 0.01, P(F/A3) = 0.005, P(F/A4) = 0.0002

What is the probability of each of these events being the cause in the event of a missile failing to reach its target?

$$P(A|B) = \frac{P(A) * P(B|A)}{\in P(B|E1) * P(E1)}$$

Prob of failure if event has occurred X prob if event occurred.

(0.3*0.0001)+(0.01*0.005)+(0.005*0.002)+(0.0002*0.04) = 0.000098

P(F/A1) = $\frac{0.3*0.0001}{0.000098} = \mathbf{0.31}$

P(F/A3) = $\frac{0.005*0.002}{0.000098} = \mathbf{0.1}$

P(F/A2) = $\frac{0.01*0.005}{0.000098} = \mathbf{0.51}$

P(F/A4) = $\frac{0.0002*0.04}{0.000098} = \mathbf{0.08}$

QUESTION 5

For a device with a failure probability of 0.02, when subjected to a specific test environment, use the binomial distribution to calculate the probabilities that a test sample of 25 devices will contain a) No failures, b) one failure and c) more than one failure.

a) $\Pr(Y = 0) = \frac{25!}{0!(25-0)!} * 0.02^0(1 - 0.02)^{25-0}$ = **0.6035**

b) $\Pr(Y = 1) = \frac{25!}{1!(25-1)!} * 0.02^1(1 - 0.02)^{25-1}$ = **0.3079**

c) $\Pr(Y > 1) = 1 - (\Pr(y = 0) + \Pr(y = 1))$ = **0.0886**

QUESTION 6

One of your suppliers has belatedly realised that about 10% of the batches of a particular component recently supplied to you has a manufacturing fault that has reduced their reliability. Batch identity has, however been maintained, so your problem is to sort the batches that have this fault (bad batches) from the rest. An accelerated test has been devised such that component from good batches have a failure probability of 0.02 whereas those from bad batches have a failure probability of 0.2. A sample plan has been devises as follows:

1 Take a random sample of 25 items from each unknown batch and subject them to the test

2 if there are 0 or 1 failed components, decide that the batch is a good one.

3 if there are two of more failures, decide that the batch is a bad on.

There are risks in this procedure, in particular there is (i) the risk of deciding that a good batch is bad and (ii) that a bad batch is good. Use Bayes theorem and your answer to question 2 to evaluate this risk.

i. **the risk of deciding that a good batch is bad**

$$\boldsymbol{P(A|B) = \frac{P(A) * P(B|A)}{\in P(B|E1) * P(E1)}}$$

P(A) batch is good

90% are good, 0.9 given that 10% has a fault.

P($\bar{A}$)=**0.1**

P(B|A)=**0.0886** (Method from Question 5)

B => Failure per batch of 25

P(B|$\bar{A}$)=**0.9726**

P(A|B) = $\frac{0.9 \times 0.0886}{(0.0886 \times 0.9)+(0.9726 \times 0.1)}$

P(A|B) = $\mathbf{0.45}$ **(A good batch)**

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ii. **A bad batch is good**

A = Bad Batch

B = 0 or 1 failure

P(A) = 0.1

P($\bar{A}$) = 0.9

$$\Pr(Y = 0) = \frac{25!}{0!(25-0)!} * \dots \dots ..^{0} \, (1 - \cdots \dots)^{25-0}$$

$\Pr(Y = 0) =$ **0.0038**

$$\Pr(Y = 1) = \frac{25!}{1!(25-1)!} * \dots \dots \dots^{1} \, (1 - \cdots \dots)^{25-1}$$

$\Pr(Y = 1) =$ **0.0236**

P(B|A) = $0.0038 + 0.0236 = \mathbf{0.0274}$

P(B|$\bar{A}$) = $0.6075 + 0.3079 = \mathbf{0.9114}$

P(A|B) = $\frac{0.1 \times 0.0274}{(0.0274 \times 0.1)+(0.9114 \times 0.9)}$

P(A|B) = $\mathbf{0.0033}$

QUESTION 7

A railway train is fitted with three engine/transmission units that can be assumed to exhibit a constant hazard with a mean life of 200 hrs. In a 15-hr. working day calculate the probability of a train having a) no failed engine/transmission units b) not more than one failed unit, c) not more than two failed units.

$$R(15) = e^{-\frac{15}{200}} = \mathbf{0.928} =' q' \text{ value}$$

$$1 - 0.928 = \mathbf{0.072} = \text{'p' value}$$

$$f(x) = \frac{n!}{x!(n-x)!} p^x q^{(n-x)}$$

a) $\Pr(Y = 0) = \frac{3!}{0!(3-0)!} * 0.072^0 * 0.928^{3-0}$ = **0.7992**

b) $\Pr(Y \geq 1) = \frac{3!}{1!(3-1)!} * 0.072^1 * 0.928^{3-1}$ = **0.1860**

1 - (0.7992+0.1860) = **0.0148**

c) $\Pr(Y \geq 2) = \frac{3!}{2!(3-2)!} * 0.072^2 * 0.928^{3-2}$ = **0.0144**

1 - (0.7992+0.1860+0.0144) = **0.0004**

QUESTION 8

Two boilers are used to supply steam for a steam turbine. Each boiler can completely supply the steam to the turbine 80% of the time.

The probability of failure of each boiler is 0.05 and that of both boilers failing simultaneously is 0.03.

Find the probability of the steam turbine getting the complete supply of steam and one of the boilers failing simultaneously.

SOLUTION:

T = turbine steam

B1 = Boiler 1 fails, B2 = Boiler 2 Fails

P(B1) = 0.05, P(B2) = 0.05, P(B1B2) = 0.03

$P(B1|B2) = \frac{P(B1B2)}{P(B2)} = \frac{0.03}{0.05} = \mathbf{0.6}$

$P(\overline{B1}|B2) = 1 - P(B1|B2) = 1-0.6 = \mathbf{0.4}$

$P(\overline{B1}B2) = P(\overline{B1}|B2)*P(B1) = 0.4*0.05 = \mathbf{0.02}$

$P(T|\overline{B1}B2)P(\overline{B1}B2) = 0.8*0.02 = \mathbf{0.016}$

Therefore probability of turbine getting steam + one boiler fail

As B1=B2 (0.05)

You can sum the probability

0.016+0.016 = **0.032**

QUESTION 9

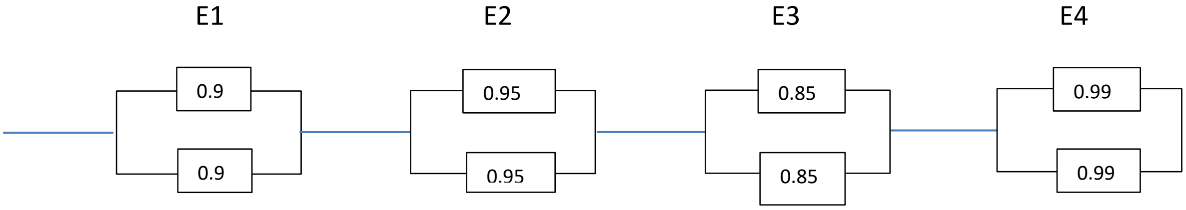
Consider a thermal power plant composed of four subsystems, as shown below.

Each subsystem consists of two identical units connected in parallel to increase its reliability.

The power plant functions and generates power only when at least one unit of each subsystem between points 1 and 2 functions.

The probability that each unit does not depends on whether or not the other units are functional.

Determine the probability that the power plant generates power.



SOLUTION:

E1 = 1 - [(1-0.9) * (1.0.9)] = **0.99**

E2 = 1 - [(1-0.95) * (1.0.95)] = **0.9975**

E3 = 1 - [(1-0.85) * (1.0.85)] = **0.9775**

E4 = 1 - [(1-0.99) * (1.0.99)] = **0.9999**

THEREOFRE - 0.99*0.9975*0.9775*0.9999 = **0.9652**

TUTORIAL SHEET 3 (RELIABILITY AND HAZARD FUNCTIONS)

QUESTION 1

The hazard function of electric motors is given by $h(t) = 5*10^{-8}t^{0.6}$ failures per hour.

If 200 electric motors are tested, how many motors are expected to fail in the first 1000 hours.

SOLUTION:

$h(t) = 5*10^{-8}t^{0.6}$ failures per hour

$$R(t) = exp^{\left[-\int_0^t h(x)dx\right]}$$

$$\boldsymbol{R(t) = e^{\left[-5\times10^{-8}\int_0^t x^{0.6}\,dx\right]}}$$

$$\left.\int_0^t x^{0.6} = \frac{x^{1.6}}{1.6} = \frac{1}{1.6}x^{1.6} = 0.625x^{1.6}\right\} \textbf{INTERGATION}$$

$$R(t) = e^{-\left(3.125\times10^{-8}\,t^{1.6}\right)}$$

$$\text{at } t = 1000 \text{ hours}$$

$$R(1000) = e^{-\left(3.125\times10^{-8}\,1000^{1.6}\right)} = \mathbf{0.99803020}$$

$$P(R) = 1 - R(1000) = 1 - 0.99803020 = 0.001969799$$

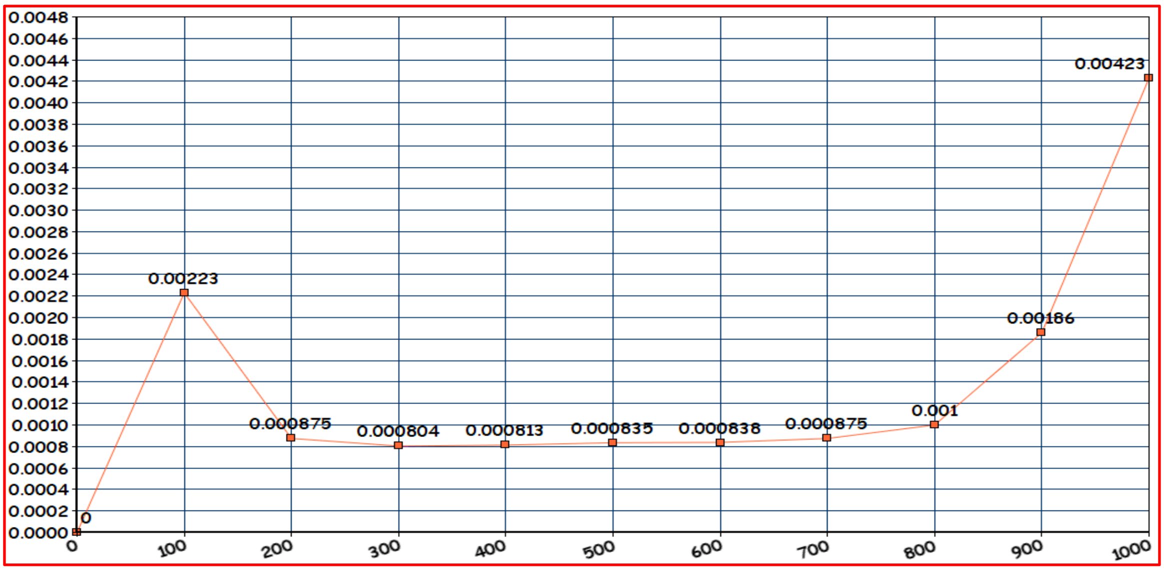
$$0.001969799 \times 200 = \mathbf{0.3939}$$

QUESTION 2

The data on the lives of 1000 identical components is given in the following table, plot the failure rate (Y) versus time curve (X):

Time Interval (100 Hours)	Number of Failures in the Interval
1	223
2	68
3	57
4	53
5	50
6	46
7	44
8	46
9	77
10	142

Time Interval	Time of Observation (Hrs)	Number of failures in Interval	Number of surviving Components (the last surviving number – number of failures in intervals)	Reliability (surviving number/1000)	Rate of change of R(t) $\frac{\Delta reliability}{\Delta time\ of\ obvervation}$	Failure Rate (R(t) – the value before in Reliability)
0	0	0	1000	1	-	-
1	100	223	777	0.777	0.00223	0.00223
2	200	68	709	0.709	0.00068	0.000875
3	300	57	652	0.652	0.00057	0.000804
4	400	53	599	0.599	0.00053	0.000813
5	500	50	549	0.549	0.00050	0.000835
6	600	46	503	0.503	0.00046	0.000838
7	700	44	459	0.459	0.00044	0.000875
8	800	46	413	0.413	0.00046	0.00100
9	900	77	336	0.336	0.00077	0.00186
10	1000	142	194	0.194	0.00142	0.00423



QUESTION 3

Find the MTTF of a component which has a reliability of 0.99 for an operating time of 1000 hours.

$$R(t) = e^{-\lambda t}$$
$$R(t) = 0.99 = e^{-\lambda(1000)}$$
$$ln0.99 = -\lambda(1000)$$
$$\lambda = 10.05 \times 10^{-6}$$

$$\int_0^{\infty} R(t)dt = \int_0^{\infty} e^{-\lambda t}dt$$
$$= (-\frac{1}{\lambda}e^{-\lambda t})_0^{\infty}$$
$$= -\frac{1}{\lambda}(e^{-\infty} - e^0)$$
$$= -\frac{1}{\lambda}(0-1) = \frac{1}{\lambda} = \frac{1}{10.05\times10^{-6}} = \mathbf{99502.487}\ \textbf{hours}$$

QUESTION 4

The reliability functions of a system is given by R(t) = e^{-t^2} Find the corresponding

a) Failure rate functions

b) Probability density function of the failure time

NOT BEEN DISCUSSED IN THE LECTURE

QUESTION 5

Find the reliability of an engine for an operating time of 500 hours if the failure rate is 4 per 10^6 hours.

QUESTION 6

If the failure rate of a component is 1 in 10^4 hours determine the following

a) the reliability function of the component

I. Reliability function of a system with four components in series
II. Reliability function of a system with four components in parallel

SOLUTION:

$h(t) = \frac{1}{10^4} = \mathbf{10^{-4}}$ failures per hour

$R1(t) = e^{-h(t)t} = \boldsymbol{e^{-10^{-4}(t)}}$

$in\ series\ R1(t)^4 = \boldsymbol{e^{-4(1\times10^{-4})t}}$

$in\ parallel\ 1-[1-R1(t)]^4 = \mathbf{1-(1-}\boldsymbol{e^{-10^{-4}t}}\mathbf{)^4}$

QUESTION 7

The life of an electronic component is found to follow exponential distribution. Determine the median life of the component. (Remember the median corresponds to the time at which 50% of the components will fail (or survive)

SOLUTION:

$$Ft(t) = 1 - e^{-\lambda t}$$
$$Ft(t = to) = 1 - e^{-\lambda to} = 0.5$$
$$e^{-\lambda to} = 0.5$$
$$-\lambda to = -0.693147$$
$$to = \frac{\mathbf{0.693}}{\boldsymbol{\lambda}}$$

QUESTION 8

The life of windshield wipers of a car are found to follow exponential distribution with a failure rate of 0.0005 failures per hour.

SOLUTIONS:

a) Find the reliability of the wipers at 2000 hours

R(t) = $e^{-\lambda t} = e^{-0.0005\times2000} = \mathbf{0.3679 = 0.370}$

b) If it is desired to have no more than 5 percent probability of failure of the wipers, how often do the windshield wipers need to be replaced? Assume the car is driven for 3 hours per day.

Find t so that R(t) = 0.05 or 5%

$$e^{-\lambda to} = 0.05$$

$$-\lambda to = -2.996$$

$$to = \frac{-2.996}{-0.0005} = \mathbf{5992\ hours}$$

QUESTION 9

The failure time of a machine component in hours of operation is known to follow the following distribution

INVALID QUESTION

Find the cumulative distribution function FT(t)

Find the reliability function R(t)

Find the failure rate function h (t)

QUESTION 10

The life of a voltmeter in hours has an exponential distribution with λ = 0.0005.

Determine the following

SOLUTIONS:

a) The probability that the voltmeter's life is at least 8000 hours

$$e^{-\lambda t} = e^{-0.0005 \times 8000} = \mathbf{0.0183}$$

b) The probability that the voltmeters life is at most 8000 hours

$$1 - 0.0183 = \mathbf{0.9817}$$

QUESTION 11

The life of a roller bearing used in a machine (in hours) follows the Weibull distribution with β = 2 and η=800 hours.

a) Determine the mean and standard deviation of the roller bearing
b) The probability that the bearing will fail before 400 hours.

SOLUTIONS:

GAMMA FUNCTION

a) **Use Gamma Function**

$$\boldsymbol{\Gamma(N) = (N-1)!}$$
$$\boldsymbol{T = N\,\Gamma\left(1+\frac{1}{\beta}\right)}$$
$$= 800\,\Gamma\left(1+\frac{1}{2}\right) \gg 800\Gamma(1.5)$$
$$= (1.5-1)! = \mathbf{0.8862}\ (will\ get\ Math\ Error\ at\ this\ point)$$

Then substitute the value

$$\mathbf{Mean} = \mathrm{n\ x}\,\Gamma(\mathrm{N})$$
$$\mathbf{Mean}\ = 800 \times 0.8862 = \mathbf{708.96}$$

MEAN

$$\boldsymbol{\sigma^2 = N^2\left\{\left[\Gamma\left(1+\frac{2}{\beta}\right)\right]-\left[\Gamma\left(1+\frac{1}{\beta}\right)^2\right]\right\}}$$

$$\sigma_T^2 = 800^2(1-0.8862^2) = 13737$$

$$\sigma = \mathbf{370.64\ hours}$$

STANDARD DEVIATION

b) The probability that the bearing will fail before 400 hours.

$$P[T \leq 400\ HOURS] = F_T(400)$$
$$\boldsymbol{F_T(t) = 1 - e^{-\left(\frac{t}{N}\right)^{\beta}}}$$
$$F_T(400) = 1 - e^{-\left(\frac{400}{800}\right)^2} = \mathbf{0.2212}$$

PROBABILITY WILL FAIL BEFORE 400 HOURS

QUESTION 12

The failure time of the tires (in terms of miles of operation) used in an automobile is found to follow a normal distribution with a mean of 40,000 and a standard deviation of 8000.

a) find the reliability of a car at 50,000 miles

$$\boldsymbol{Z = \frac{X - mean}{Standard\ Dev.}}$$

$$P = \frac{50000 - 40000}{8000} = \mathbf{1.25}$$
$$z = 1.25 \ \gg Ri = \mathbf{0.10565}$$

b) Ignoring all other failure modes, find the reliability of the automobile considering all the four tires at 50,000 miles.

$$\boldsymbol{Ra = \prod^4 (Ri)} = Ri^4 = 0.1056^4 = \mathbf{0.000124}$$

c) If it desired to have no more than 5% probability of failure of the automobile how often do the tires need to be replaced. Assume the car is driven on average 250 miles a day.

SOLUTION:

Because 250 miles a day = ¼ a day

$$(0.95)^{\frac{1}{4}} = 0.987258$$
$$1 - 0.987258 = 0.0127428\ (Based\ on\ Z\ table)$$

From Z table = 2.23

$$z = \frac{y - n}{\sigma} \gg 2.23 = \frac{y - 40000}{8000}$$
$$t = \frac{57840}{250} = \boldsymbol{232\ days}$$

QUESTION 13

A series system consists of 5 components with failure rates of 0.0001, 0.0003, 0.0002, 0.0005 and 0.0008.

Find the reliability at 100 hours.

If it is desired to increase the 100-hour reliability by 5%, select a single component and its new failure rate to accomplish this under the following conditions

SOLUTION:

$$\boldsymbol{R = e^{-\lambda t}}$$

$R_1 = 0.99$, $R_2 = 0.97$, $R_3 = 0.98$, $R_4 = 0.95$, $R_5 = 0.92$
$R_T =$ **0.82**

a) when no cost constraints

Desired to increase the 100-hour reliability by 5%,

0.82 X 1.05 = 0.861

$$\Delta R = 0.861 - 0.82 = \mathbf{0.0413}$$

$$Comp\ 1 \gg \Delta R_1 \times (R_2 \times R_3 \times R_4 \times R_5) = 0.0413$$
$$\Delta R_1 \times (0.97 \times 0.98 \times 0.95 \times 0.92) = 0.0413$$
$$\Delta \boldsymbol{R_1} = 0.0495$$

Find for $\Delta R_2, \Delta R_3, \Delta R_4, \Delta R_5$

$$\Delta \boldsymbol{R_2} = 0.0485\ \Delta \boldsymbol{R_3} = 0.049\ , \Delta \boldsymbol{R_4} = 0.0476\ , \Delta \boldsymbol{R_5} = 0.0462$$
$$\Delta R_T = R_5 + \Delta R_5 = 0.92 + 0.0462 = \mathbf{0.96447}$$
$$0.96447 = e^{-\lambda(100)}$$
$$\ln(0.96447) = -\lambda(100)$$
$$\lambda = \mathbf{0.0003618}$$

Choose the lowest reliability from $\Delta \boldsymbol{R_1} - \Delta \boldsymbol{R_5}$

b) when a unit reliability improvement costs 500, 400, 600, 1000 and 800 for components 1, 2 3 4 and 5.

$$\Delta R_1 = 0.0495 \times 500 = 24.75$$
$$\Delta \boldsymbol{R_2} = \mathbf{0.0485 \times 400 = 19.4}$$
$$\Delta R_3 = 0.049 \times 600 = 29.4$$
$$\Delta R_4 = 0.0476 \times 1000 = 47.6$$
$$\Delta R_5 = 0.0462 \times 800 = 36.99$$

From $\Delta \boldsymbol{R_1} - \Delta \boldsymbol{R_5}$, multiply with those improvement costs, choose the lowest.

QUESTION 14

A series system is component of 5 components with failure rates of 0.0005, 0.0001, 0.0008, 0.004 and 0.0002 per hour. Find the reliability at 200 hours.

$$R = e^{-\lambda t}$$

$$R_1 = 0.905$$
$$R_2 = 0.980$$
$$R_3 = 0.852$$
$$R_4 = 0.44$$
$$R_5 = 0.96$$
$$\boldsymbol{R_T = 0.32}$$

QUESTION 15

Ten identical component are connected in parallel to achieve a system reliability of 0.95, determine the reliability of each component

$$R = 1 - [(1-R)^{10}] = 0.95$$

$$-[(1-R)^{10}] = 0.95 - 1$$

$$-[(1-R)^{10}] = -0.05$$

$$[(1-R)^{10}] = 0.05$$

$$(1-R) = 0.05^{\frac{1}{10}}$$

$$(1-R) = 0.741$$

$$-R = 0.741 - 1$$

$$-R = -0.259$$

$$R = 0.259$$

$$R = \mathbf{0.26}$$

QUESTION 16

A Parallel system is components of 5 components with failure rates of 0.009, 0.007, 0.006, 0.005 and 0.008 per hour

Find the reliability of the system at 100 hours.

If it is desired to increase the reliability of the system by 2% and its new failure rate to accomplish this under the following conditions.

R_1=0.407, R_2=0.497, R_3=0.549, R_4=0.607, R_5=0.449

$$1-[0.593\times 0.503\times 0.451\times 0.393\times 0.551]=1-0.0292$$
$$=\mathbf{0.9708}$$

a) **when no cost constraints)**

$$\mathbf{0.9708\times 0.02=0.0194} \quad \textbf{OR} \quad \mathbf{\Delta R=0.98-0.97=0.0194}$$

$$\boldsymbol{dR=\frac{dR_o}{\prod_{i=1}^{n}(1-Ri)}}$$

$$\vdots\ \boldsymbol{dR}=individual\ component\ ,\boldsymbol{dR_o}=\Delta R_o\ overall\ components\ change\ R,$$
$$\boldsymbol{Ri}=individual\ comp\ not\ including\ one\ being\ examined$$

$$dR_1=\frac{0.0194}{0.503\times 0.451\times 0.393\times 0.551}=0.395$$

Find for dR_2, dR_3, dR_4, dR_5

dR_2=0.335, dR_3=0.300, **dR_4=0.262**, dR_5=0.367

Choose the lowest reliability from $\boldsymbol{dR_1-dR_5}$

$$\frac{1-Ro}{1-R4}=\frac{1-0.9708}{1-0.3935}=0.481$$

Choose the total reliability system and the largest from $\boldsymbol{dR_1-dR_5}$

Actually you should do all $\boldsymbol{\frac{1-Ro}{1-R1},\frac{1-Ro}{1-R2},\frac{1-Ro}{1-R3},\frac{1-Ro}{1-R4},\frac{1-Ro}{1-R5}}$

b) **when a unit reliability improvement costs 500, 400, 600, 1000 and 800 for components 1, 2 3 4 and 5.**

dR_1 X 500 = 197.5, **dR_2X 400=134**, dR_3X 600= 180, dR_4X 1000= 262, dR_5X 800= 293.6

TUTORIAL SHEET 4 (PLOTTING DISTRIBUTION)

QUESTION 1

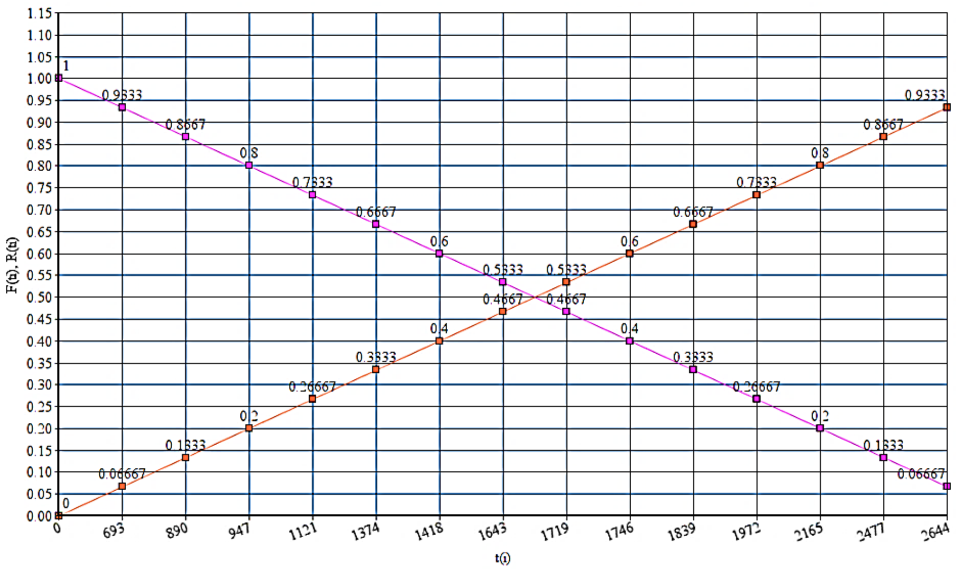
The following failure times were observed for 15 hydraulic cylinders used in aircraft landing gears.

1719	1972
693	2477
1374	890
2841	1418
1121	1643
1839	2644
947	2165
1746	

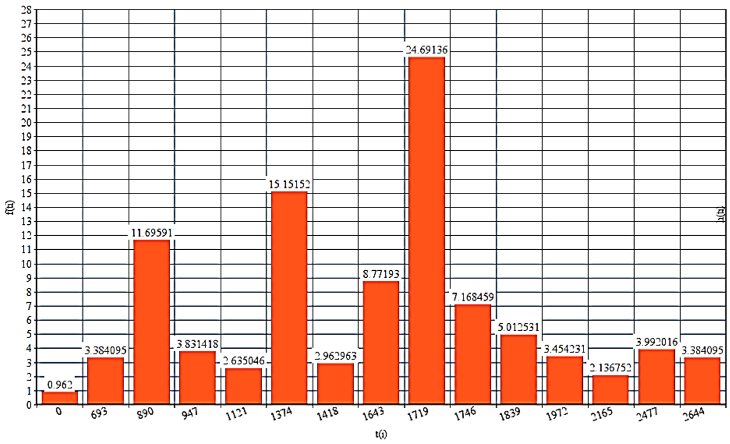
Plot the probability density, probability distribution, reliability and hazard functions of the life of the hydraulic cylinders.

Please make sure that failure times in the table given in ascending order before proceeding to any calculations.

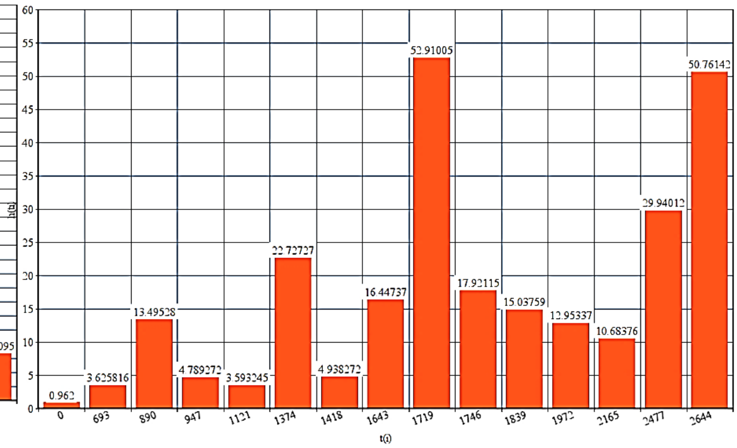
i	ti	T(i+1) - ti	F(ti) = $\frac{i}{n}$	R(ti) = $\frac{n-1}{n}$	f(ti)x10^4=$\frac{1}{n(T(i+1)-ti)}$	H(ti) x10^4 = $\frac{f(ti)}{R(ti)}$
0	0	693	0	1	0.9620	0.962
1	693	197	0.066667	0.933333	3.384095	3.625816
2	890	57	0.133333	0.866667	11.69591	13.49528
3	947	174	0.2	0.8	3.831418	4.789272
4	1121	253	0.266667	0.733333	2.635046	3.593245
5	1374	44	0.333333	0.666667	15.15152	22.72727
6	1418	225	0.4	0.6	2.962963	4.938272
7	1643	76	0.466667	0.533333	8.77193	16.44737
8	1719	27	0.533333	0.466667	24.69136	52.91005
9	1746	93	0.6	0.4	7.168459	17.92115
10	1839	133	0.666667	0.333333	5.012531	15.03759
11	1972	193	0.733333	0.266667	3.454231	12.95337
12	2165	312	0.8	0.2	2.136752	10.68376
13	2477	167	0.866667	0.133333	3.992016	29.94012
14	2644	197	0.933333	0.066667	3.384095	50.76142
15	2841		1	0		



Graph f(ti)



Graph h(ti)



QUESTION 2

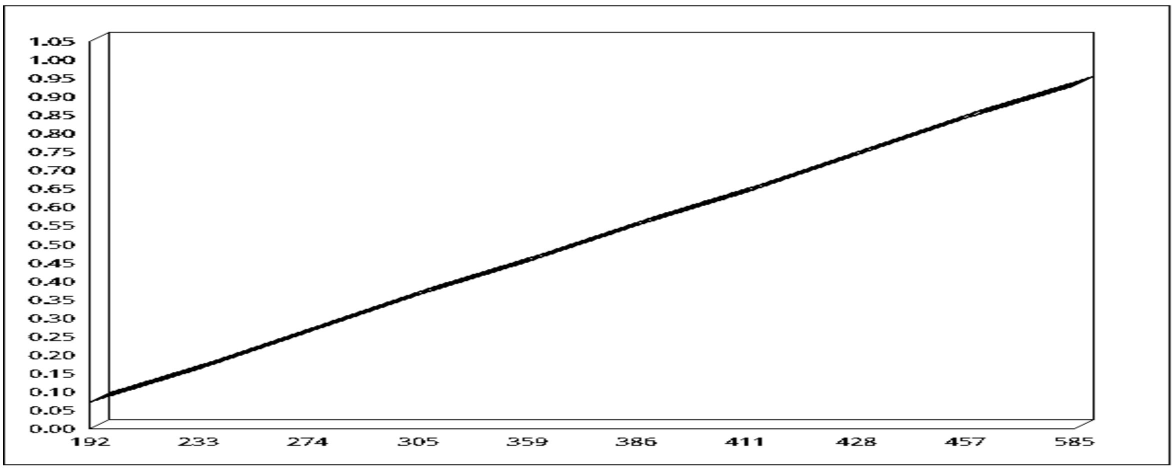
A sudden death test is conducted by dividing randomly a group of 50 valves used in the ice making unit of household fridges into groups of 5 each.

The failure times observed for the first unit of each of the 10 groups are given by

192, 233, 274, 305, 359, 386, 411, 428, 457, 585 hours.

Find the median life of the population of the valves.

i	Ti	Fi = $\frac{i-0.3}{n+0.4}$
1	192	0.07
2	233	0.16
3	274	0.26
4	305	0.36
5	359	0.45
6	386	0.55
7	411	0.64
8	428	0.74
9	457	0.84
10	585	0.93



QUESTION 3

The tensile strengths of 12 welded joints have been observed to be

31.325, 31.556, 31.234, 31.998, 31.404, 31.263, 31.440, 31.785, 31.492, 31.357, 31.285, 31.643.

Plot the data assuming it follows an exponential distribution.

i	x-axis ti	$F(ti) = \frac{i}{n+1}$	y-axis = $\frac{1}{1-F(ti)}$	$f(ti) = \frac{i-0.3}{n+0.4}$	Y-axis = $\frac{1}{1-f(ti)}$
1	31.234	0.08	1.08	0.06	1.06
2	31.263	0.15	1.18	0.14	1.16
3	31.285	0.23	1.30	0.22	1.28
4	31.325	0.31	1.44	0.30	1.43
5	31.357	0.38	1.63	0.38	1.61
6	31.404	0.46	1.86	0.46	1.85
7	31.44	0.54	2.17	0.54	2.18
8	31.492	0.62	2.60	0.62	2.64
9	31.556	0.69	3.25	0.70	3.35
10	31.643	0.77	4.33	0.78	4.59
11	31.785	0.85	6.50	0.86	7.29
12	31.998	0.92	13.00	0.94	17.71

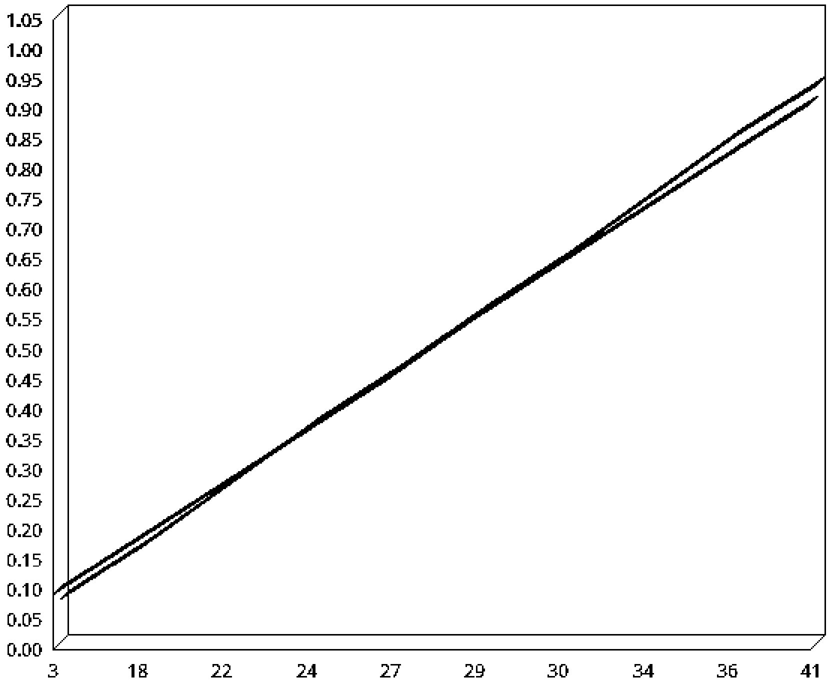
*Remember not to join the dots, try to find the best fit.

To find the slope, choose 2 points from Y axis - $\frac{lny_2 - lny_1}{x_2 - x_1}$

QUESTION 4

The crushing strengths of 10 welded joints have been observed to be 18, 27, 24, 30, 22, 36, 41, 3, 29 and 34. Plot the data according to a normal distribution.

i	ti	y-axis F(ti)		y-axis f(ti)	
1	3	0.09	1.10	0.07	~~1.07~~
2	18	0.18	1.22	0.16	~~1.20~~
3	22	0.27	1.38	0.26	~~1.35~~
4	24	0.36	1.57	0.36	~~1.55~~
5	27	0.45	1.83	0.45	~~1.82~~
6	29	0.55	2.20	0.55	~~2.21~~
7	30	0.64	2.75	0.64	~~2.81~~
8	34	0.73	3.67	0.74	~~3.85~~
9	36	0.82	5.50	0.84	~~6.12~~
10	41	0.91	11.00	0.93	~~14.86~~



QUESTION 5

A system is components of five identical components each having a failure rate of 0.001 per hour.

Find the reliability of the system at 100 hours and 200 hours if at least two of the components must operate for the survival of the system.

$$R(t) = e^{-\,0.001\times 100} = \mathbf{0.99005}$$
$$1 - 0.99005 = \mathbf{0.00995}$$

New Formula

$$\boldsymbol{R_{sys} = \sum_{r=2}^{5} \binom{n}{r} (e^{-\lambda t})^{r} \, (1 - e^{-\lambda t})^{n-r}}$$

$$1 - \sum_{0}^{1} \binom{n}{0} (0.99005)^{0} \, (1 - 0.99005)^{5-0} - \binom{n}{1} (0.99005)^{1} (1 - 0.99005)^{5-1}$$

$$\binom{n}{0} = \binom{5}{0} \gg \frac{5!}{0!\,5!} = 1 \qquad \binom{n}{1} = \binom{5}{1} \gg \frac{5!}{1!\,(5-1)!} = 5$$

$$\boldsymbol{R_{sys}} = \mathbf{0.999999951}$$

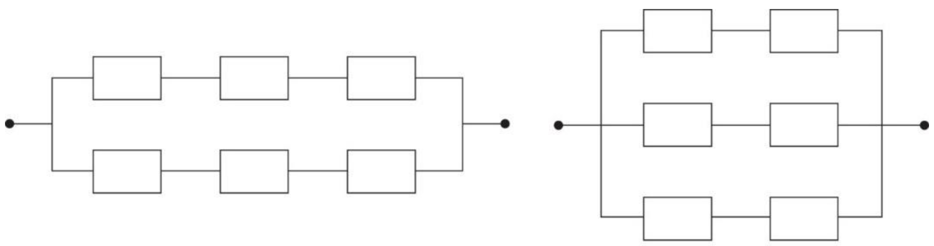
Reliability for 100 Hours

$$find \ when \ t = 200 \ hours \ ; \textit{same method}$$

QUESTION 6

Six identical components are connected in two different ways shown below.

Identify the configuration which yields a higher value of reliability.



(a) (b)

$$R_{sys}A = 1 - [(1-R)^2] = 1 - [(1 - Ro^3)^2] = 1 - (1 + Ro^6 - 2Ro^3)$$
$$= 2Ro^3 - Ro^6$$

$$\boldsymbol{R_{sys}B = 1 - [(1-R)^3] = 1 - [(1 - Ro^2)^3] = 1 - (1 - 3Ro^2 + 3Ro^4 - Ro^6)}$$
$$\boldsymbol{= 3Ro^2 - 3Ro^4 + Ro^6}$$

B is more reliable as it with $\boldsymbol{3Ro^2}$ **lead**

TUTORIAL SHEET 5

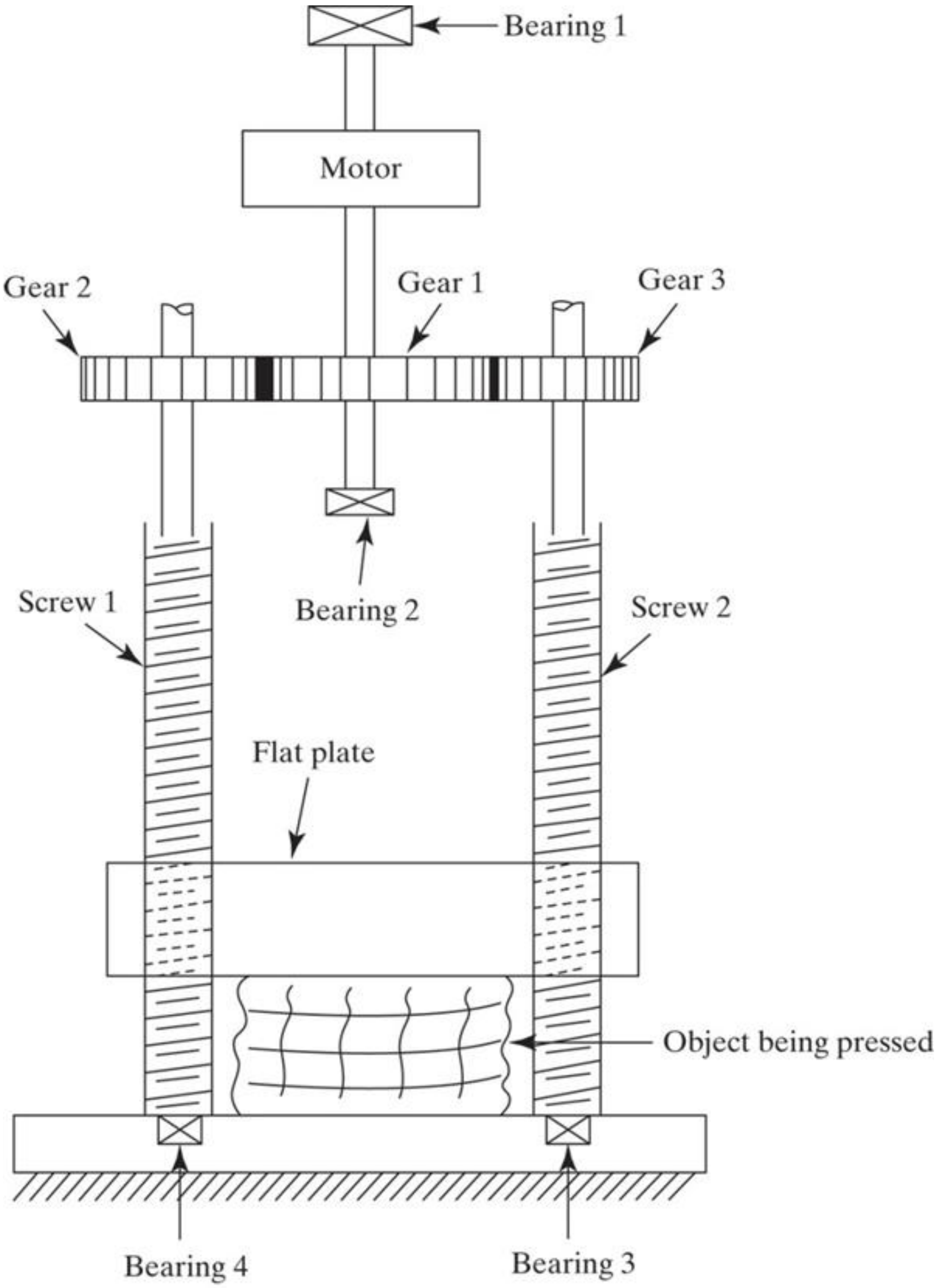
QUESTION 1

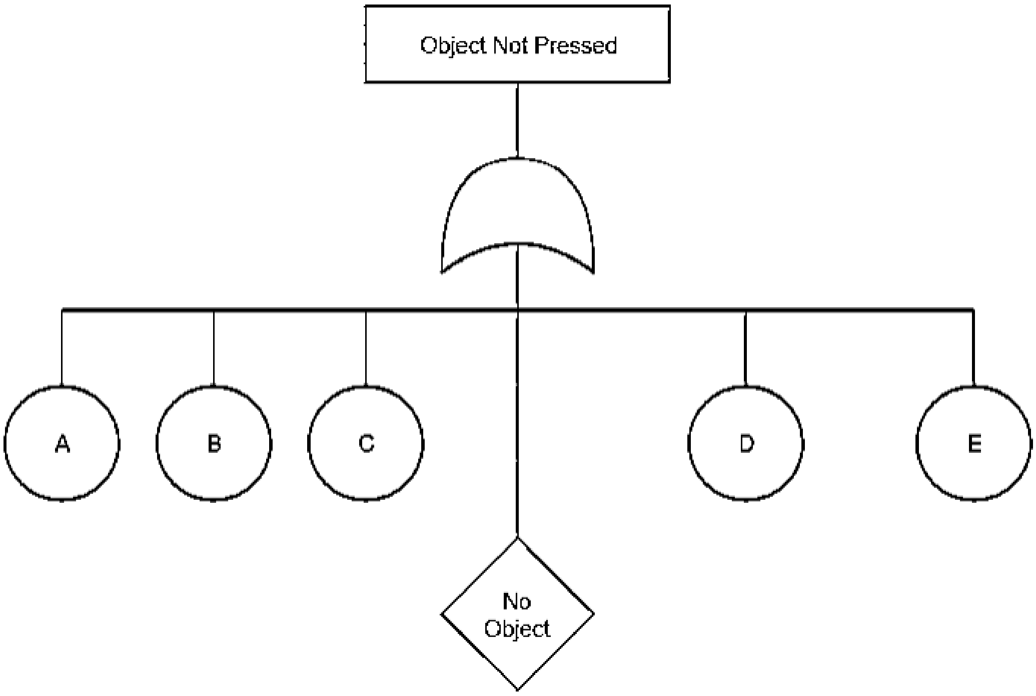
A mechanical Press consists of four bearings, a motor, three gears, two screws and a flat plate.

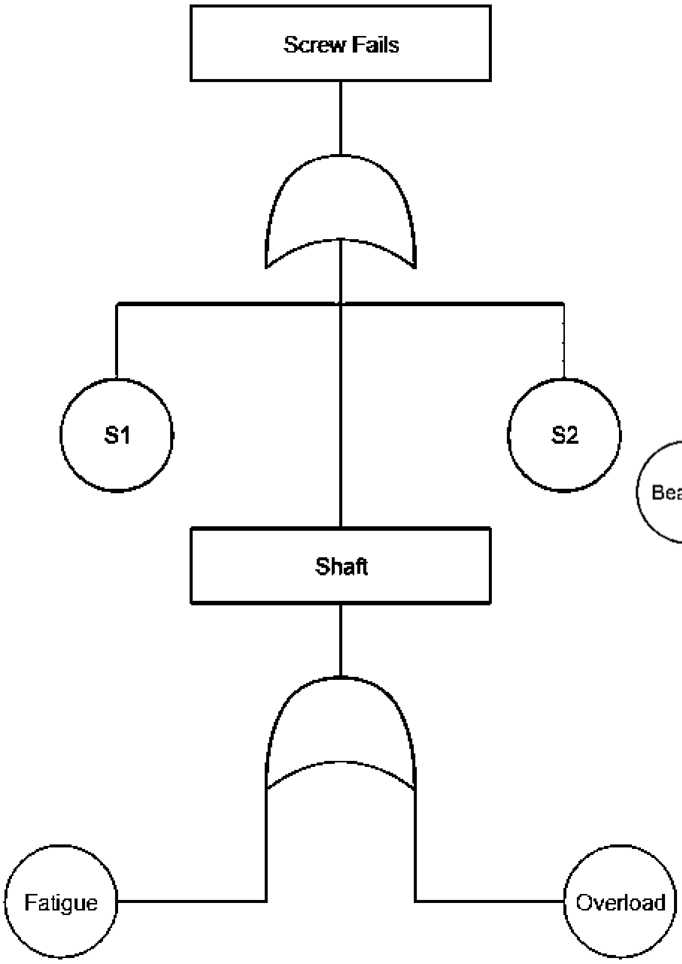
If the TOP event is “object not pressed” construct a fault tree of the system.

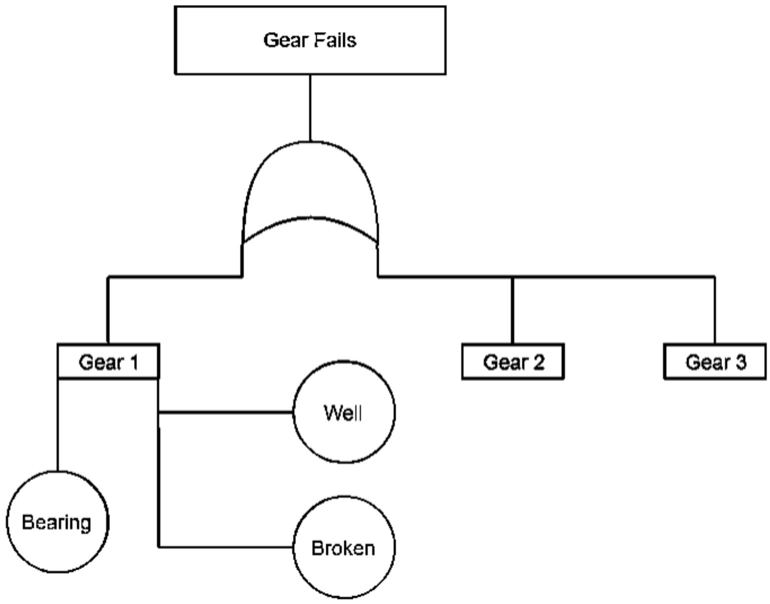
QUESTION 2

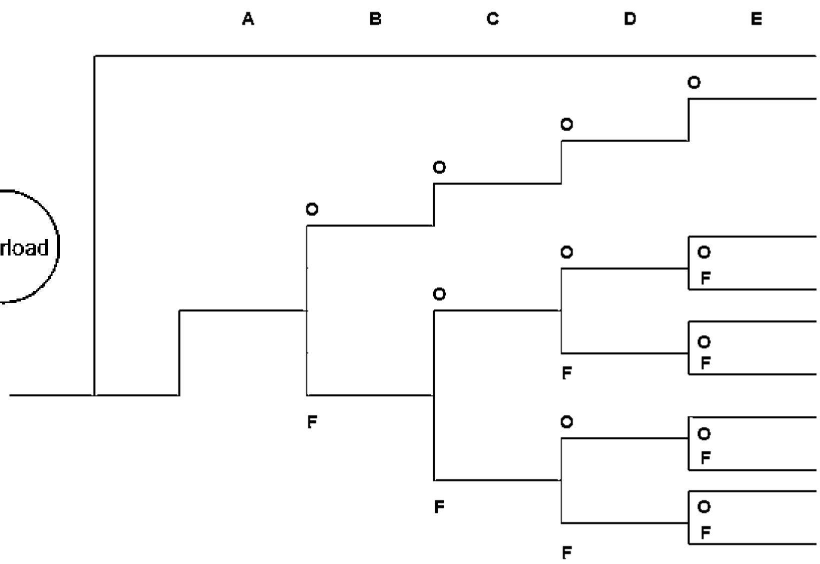
Construct an event tree for the mechanical press above.




Object Not Pressed
A
B
C
D
E
No Object


Screw Fails
S1
S2
Shaft
Fatigue
Overload


Gear Fails
Gear 1
Gear 2
Gear 3
Well
Bearing
Broken


A
B
C
D
E
O
F

TUTORIAL SHEET 6

QUESTION 1

A manufacturer sells lawnmowers with a 6-month free replacement warranty.

The failure time of the lawn mower is known to follow an exponential distribution with a failure rate of 0.02 per month.

The manufactures cost of production of a lawn mower is €200.

If the lawnmowers are production in a lot size of 10,000 units determine the total amount of warranty reserve fund to be kept aside by the manufacturer.

$$C_w = N\bar{c}_W = Nc_1(1 - e^{-\frac{t\omega}{\mu}})$$

Information Given

c_1 = €200

t_ω = 6 months

λ = 0.02 = $\frac{1}{\mu}$ $\rightarrow$ $\mu = 50\ months$

N = 10,000

Substitute into formula

$$C_w = Nc_1(1 - e^{-\frac{t\omega}{\mu}})$$

$$C_w = 10{,}000(200)(1 - e^{-\frac{6}{50}})$$

$$C_w = €226\,159.1266$$

QUESTION 2

A manufacturer sells lawnmowers with a 6-month pro rata warranty.

The failure time of the lawn mower is known to follow an exponential distribution with a failure rate of 0.02 per month.

The manufactures cost of production of a lawn mower is €200.

If the lawnmowers are production in a lot size of 10,000 units determine:

a) Cost of the lawn mower to the manufacturer without the warranty cost
b) Amount of warranty reserve fund to be kept aside by the firm for the whole batch of lawn mowers.

$$\frac{\bar{C}_w}{C_1} = 1 - \frac{\mu}{t\omega}(1 - e^{-\frac{t\omega}{\mu}})$$

Information Given

c_1 = €200

t_ω = 6 months

λ = 0.02 = $\frac{1}{\mu}$ $\rightarrow$ $\mu = 50\ months$

N = 10,000

Substitute into formula

$$\frac{\bar{C}_w}{C_1} = 1 - \frac{\mu}{t\omega}(1 - e^{-\frac{t\omega}{\mu}})$$

$$\frac{\bar{C}_w}{C_1} = 1 - \frac{50}{6}(1 - e^{-\frac{6}{50}})$$

$$\frac{\bar{C}_w}{200} = 0.057670305$$

$$\bar{C}_w = 0.057670305\,(200)$$

$$\bar{C}_w = €\,11.5340612$$

a) €200 – €11.5340612 = **€188.4659388**
b) 10,000 x €11.5340612 = **€11 340.612**

QUESTION 3

In Question 2, the manufacturer considers to offer a lump sum warranty plan equivalent to the pro-rata plan by offering a lump sum to its customers whose mowers fail within the warranty period. Find the percentage of the price to be refunded to customers whose lawn mowers fail before the expiration of the warranty period.

$$\overline{C}_{\omega} = \frac{C_1}{t\omega_2 - t\omega_1}\left[(t\omega_2 - t\omega_1) + \mu\left(e^{-\frac{t\omega_2}{\mu}} - e^{-\frac{t\omega_1}{\mu}}\right)\right]$$

Information Given

- c_1 = €200
- t_ω = 6 months
- λ = 0.02 = $\frac{1}{\mu}$ $\rightarrow \mu = 50\ months$
- N = 10,000

Substitute into formula

$$\overline{C}_{\omega} = \frac{C_1}{t\omega_2 - t\omega_1}\left[(t\omega_2 - t\omega_1) + \mu\left(e^{-\frac{t\omega_2}{\mu}} - e^{-\frac{t\omega_1}{\mu}}\right)\right]$$

$$\overline{C}_{\omega} = \frac{200}{6-1}\left[(6-1) + 50\left(e^{-\frac{6}{50}} - e^{-\frac{1}{50}}\right)\right]$$

$$\overline{C}_{\omega} = 40[(5) + 50\,(0.886920436 - 0.980198673\,)]$$

$$\overline{C}_{\omega} = \text{€}\ \mathbf{13.443526}$$

10,000 X € 13.443526 = **€ 134435.26**

QUESTION 4

A manufacturer sells lawnmowers with a one-month free replacement free warranty followed by a 5 month pro rata warranty.

The failure time of the lawn mower is known to follow an exponential distribution with a failure rate of 0.02 per month.

The manufactures cost of production of a lawn mower is €200.

If the lawnmowers are production in a lot size of 10,000 units determine the amount of warranty reserve fund to be kept aside by the manufacture for the whole batch of lawnmowers.

Information Given

- c_1 = €200
- t_ω = 6 months
- $t_{\omega 1}$ = 1 months
- $t_{\omega 2}$ = 5 months
- λ = 0.02 = $\frac{1}{\mu}$ $\rightarrow \mu = 50\ months$
- N = 10,000

Formula used:

$$t_\omega = \frac{1}{\lambda}\, ln \left\{\frac{\lambda(t_{\omega 2} - t_{\omega 1})}{e^{-\lambda t_{\omega 1}} - e^{-\lambda t_{\omega 2}}}\right\}$$

$$t_\omega = \frac{1}{0.02}\, ln \left\{\frac{0.02(6-1)}{e^{-0.02(1)} - e^{-0.02(6)}}\right\}$$

$$t_\omega = \mathbf{3.4\ months}$$

QUESTION 5

In Question 4, find the length of the free replacement warranty that is equivalent to the combined free replacement and pro-rata warranty. The relevant data are a failure rate of 0.02m tw1 = 1 month and tw2 = 6 months.

Information Given

- c_1 = €200
- t_ω = 6 months
- λ = 0.02 = $\frac{1}{\mu}$ $\rightarrow$ $\mu = 50\ months$
- N = 10,000

Lump – Sum Formula Used:

$$a = \frac{1}{1 - e^{-\frac{t\omega}{\mu}}} - \frac{\mu}{t_\omega}$$

$$a = \frac{1}{1 - e^{-\frac{6}{50}}} - \frac{50}{6}$$

$$a = 0.5099976 \text{ or } 51\%$$

QUESTION 6

A manufacturer sells lawnmowers with a 6-month free replacement warranty.

The failure time of the lawn mower is known to follow an exponential distribution with a failure rate of 0.02 per month.

The manufactures cost of production of a lawn mower is €200.

By considering the time value of money with a discounting factor of 0.05, determine the expected amount of warranty payment to be made by the manufacturer per unit.

Information Given

- c_1 = €200
- t_ω = 6 months
- λ = 0.02 = $\frac{1}{\mu}$ $\rightarrow$ $\mu = 50\ months$
- δ= 0.05

Use formula Free Replacement Warranty (FRW) with discounting factor

$$\overline{C}_\omega = \frac{c_1\lambda}{\lambda + \delta}\left\{1 - e^{-(\lambda+\delta)\,t_\omega}\right\}$$

$$\overline{C}_\omega = \frac{200(0.02)}{0.02 + 0.05}\{1 - e^{-(0.02+0.05)\,(6)}\}$$

$$\overline{C}_\omega = €19.597$$

QUESTION 7

A manufacturer sells lawnmowers with a 1-year pro rata warranty.

The failure time of the lawn mower is known to follow an exponential distribution with a failure rate of 0.02 per month.

The manufactures cost of production of a lawn mower is €200.

By considering the time value of money with a discounting factor of 0.05, determine the expected amount of warranty payment to be made by the manufacturer per unit.

Information Given

- c_1 = €200
- t_ω = 12 months
- λ = 0.02 = $\frac{1}{\mu}$ $\rightarrow$ $\mu = 50\ months$
- δ= 0.05

Use Pro Rata Warranty (PRW) Policy Formula

$$\overline{C}_\omega = \frac{c_1 \lambda}{\lambda + \delta}\{1 - e^{-(\lambda+\delta)\,t_\omega}\}$$

$$\overline{C}_\omega = \frac{200(0.02)}{0.02 + 0.05} - \frac{200(0.02)}{12(0.02 + 0.05)^2}\{1 - e^{-(0.02+0.05)\,(12)}\}$$

$$\overline{C}_\omega = 57.142857 - 68.02721\{0.568289476\}$$

$$\overline{C}_\omega = \textbf{€18.483}$$

QUESTION 8

The failure rate of an electrical appliance follows an exponential distribution with a failure rate of 0.02 per month. The manufacturer sells the applicant with a free replacement warranty for a period of one year. If the unit cost of the appliance to the manufacturer is 100 find the following

a) Expected Unit warranty cost to the manufacturer
b) Variance of the unit warranty cost to the manufacturer.

Information Given

- t_ω = 12 months
- λ = 0.02 = $\frac{1}{\mu}$ $\rightarrow$ $\mu = 50\ months$
- $c_1 + c_\omega = 100$

$$c_\omega = c_1(1 - e^{-\lambda t_\omega})$$

$$c_\omega = c_1(1 - e^{-0.02(12)})$$

$$c_\omega = c_1(0.23134)$$

Substitute c_ω into equation of $c_1 + c_\omega = 100$

$$c_1 + c_1(0.23134) = 100$$

$$c_1(1 + 0.23134) = 100$$

$$c_1 = \frac{100}{(1 + 0.23134)}$$

$$c_1 = €82.41\,;$$

$$c_\omega = €100 - €82.41$$

$$c_\omega = €17.59$$

QUESTION 9

The cost of an electrical applicant to the manufacture is €200, the failure rate of the appliance is known to be 0.25 per year. The warranty cost to the manufacturer for the failure of each appliance during the useful life of the appliance is estimated to be €200. The cost to the manufacturer to market the appliance without warranty is €1000 and this cost is expected to decrease according to the relation:

$$B_w(t_w) = \$(1000 - 500\,t_w + 100\,t_w^2)$$

Assuming that the failure follows exponential distribution find the optimal warranty period of the appliance.

Information Given

c_1 = €200

t_ω = 12 months

$\lambda = 0.25 = \frac{1}{\mu} \rightarrow \mu = 4\ months$

$M(t) = \lambda t_\omega$

$C_\omega = C_1 m\,(t_\omega) \Rightarrow C_1\lambda(t_\omega)$

$B_\omega(t_\omega) = \$\,(1000 - 500t_\omega + 100\,t_\omega{}^2)$

Total = $C_1\lambda(t_\omega) + 1000 - 500t_\omega + 100\,t_\omega{}^2$

$$\frac{d}{d_{total}} = C_1\lambda + 0 - 500 + 200\,t_\omega$$

$$t_\omega = \frac{500 - C_1\lambda}{200}$$

$$t_\omega = \frac{500 - 200(0.25)}{200}$$

$$t_\omega = 2.25\ \text{years}$$

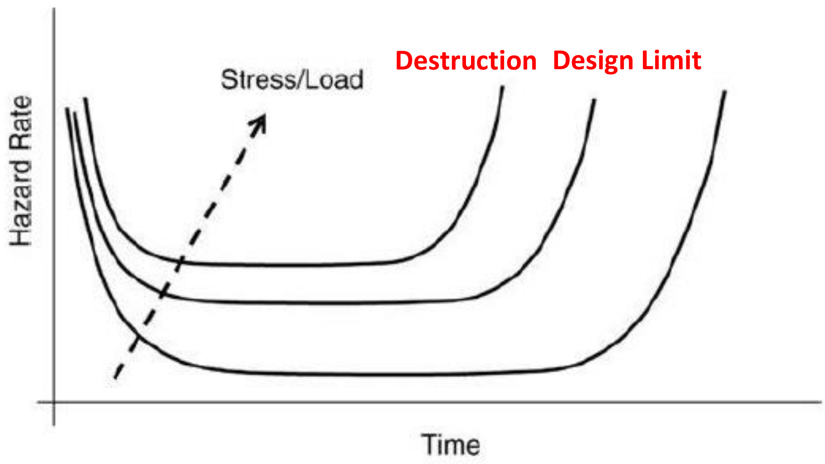
TUTORIAL SHEET 7

QUESTION 1

Discuss what happens to the bathtub curve, when stress levels approach design limits?

What happens to the bathtub curve pattern when stress reaches destruct limits?

- It also helps lower the cost of development by reducing test time and thus helping to deliver products to market in the shortest possible time
- The theoretical concept of the effect of accelerated test on a products life is shown below **higher stress levels shorten the expected product life and increase the expected failure rate at all phases of the bathtub curve**
- Bathtub become shorter and steeper when stress reaches destruct limits.



QUESTION 2

You are developing a gear box for a wind turbine power generator for an off-shore installation (sea/Ocean water). What kind of environments would you consider including in your test flow? Which test can be run sequentially, and which ones can be done in parallel?

Answer for Question 2(i):

1. Corrosion
2. Vibration
3. Temperature
4. Wind Test
5. pH of the liquid
6. Salt Water Test

Answer for Question 2(ii):

1. Corrosion and salt water test in parallel (Temperature Cycling)
2. Vibration and temperature at the same time in parallel
3. Voltage Transients including electrostatic discharge (ESD)

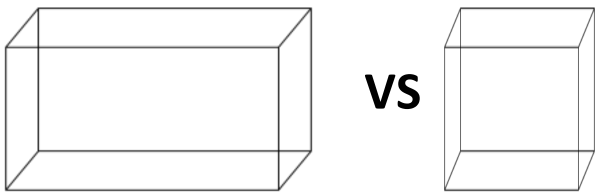
QUESTION 3

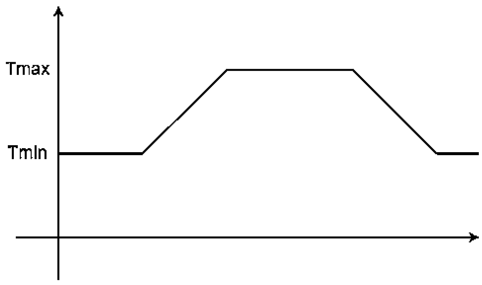
You are developing a test plan for a temperature cycling test of 300 cycles of T min, T max. How would you take into account the size and weight of your product when determining the duration of temperature dwells at Tmin and Tmax and the transition time between them?

SOLUTIONS:

Things need to consider the size and weight of your product are:

1. Frequency and how long it takes for Tmax
2. How large the unit itself.





- Both shapes will get feedback
- The big size will slower in heating and cooling temperature.
- The small size will faster in heating and cooling temperature.

QUESTION 4

Give examples of “foolish failures” and discuss ways to avoid these types of failures in testing.

- Increasing the stress beyond the products design limits may precipitate failures which would not be representative of the field environment (Foolish Failures).
- For example, plastic parts may exceed their glass transition points or even melt at high ambient temperatures, something which could not happen under normal usage conditions.
- These types of failures ae often referred as ‘foolish failures’ and should be avoided during the product testing.
- Accelerated stress levels should be chosen that so they accelerate the ‘realistic’ failure modes, which are expected in the field.

QUESTION 5

Briefly describe the concept of combined environmental reliability testing (CERT).

What are the main environmental stresses you may consider in planning a CERT for

(i) a dishwasher electronic controller
(ii) a communications satellite electronic module
(iii) an industrial hydraulic pump.

SOLUTIONS:

(i) a dishwasher electronic controller
Water, Temperature
(ii) a communications satellite electronic module
Temperature, Corrosion, Acceleration, G- Force, Radiation
(iii) an industrial hydraulic pump.
Temperature, Fluid, Vibration, Mechanical Shock